Random Walks, Scaling Limits and Criticality

Herrsching, May 20–24, 2024

ABSTRACTS

MONDAY

Quenched local limit theorem for random conductance models with long-range jumps

Takashi Kumagai (Waseda U.)

We establish the quenched local limit theorem for reversible random walk on $\mathbb{Z}^d, d \geq 2$ among stationary ergodic random conductances that permit jumps of arbitrary length. Our proof is based on the weak parabolic Harnack inequalities and on-diagonal heat kernel estimates for long-range random walks on general ergodic environments. As a byproduct, we prove the maximal inequality with an extra tail term for long-range reversible random walks, which in turn yields the everywhere sub-linear property for the associated corrector. This is a joint work with Xin Chen (Shanghai) and Jian Wang (Fuzhou).

Convergence of the Aldous-Broder chain on the high dimensional torus

Anita Winter (U. Duisburg-Essen)

The CRT is the scaling limit of the UST on the complete graph. The Aldous Broder chain on a connected regular graph $G = (V, E)$ is a MC with values in the space of rooted trees with vertices in V that has the UST on G as its invariant distribution. In Evans, Pitman and Winter (2006) the so-called root growth with regrafting process (RGRG) was constructed. It was further shown that the suitable rescaled Aldous Broder chain on the complete graph converges to the RGRG weakly with respect to the GH-topology. It was shown in Peres and Revelle (2005) that (up to a dimension depending constant factor) the CRT is also the Gromov-weak scaling limit of the UST on the d-dimensional torus, $d \geq 5$. This result was recently strengthens in Archer, Nachmias and Shalev (2024) to convergence with respect to the GH-weak topology, and therefore also with respect to the GH-topology. In this talk we show that the suitable rescaled Aldous Broder chain on the high-dimensional torus also converges to the RGRG weakly with respect to the GH-topology when initially started in the trivial rooted tree. (Joint work with Osvaldo Angtuncio Hernandez and Gabriel Berzunza Ojeda).

Distribution of the random walk conditioned on survival among quenched Bernoulli obstacles

Ryoki Fukushima (Tsukuba U.)

Consider a simple symmetric random walk that is conditioned to stay on a supercritical percolation cluster up to a large time n. Following a series of works of Sznitman in 1990s, it has recently been shown by Ding and Xu that with high probability, the random walk will be localized in a ball of volume proportional to $logn$. In this talk, I present the further refinements: (1) this ball is free of obstacles, (2) the limiting one-time distributions of the random walk are obtained. This talk is based on a joint work with Jian Ding, Rongfeng Sun and Changji Xu.

Sharp asymptotics of the disconnection time of large cylinders by simple and biased random walks

Xinyi Li (PKU)

We consider the asymptotic disconnection time of a discrete cylinder $(\mathbb{Z}/N\mathbb{Z})^d \times \mathbb{Z}$, $d \geq 2$ by simple and biased (in the $\mathbb{Z}$ direction) random walks. For simple random walk, we derive a sharp asymptotic lower bound that matches the upper bound from [A.-S. Sznitman, Ann. Probab., 2009] which allows us to identify the weak limit of the rescaled disconnection time. For the biased walk, we obtain bounds that asymptotically match in the principal order when the bias is not too strong, which greatly improves previous results from [D. Windisch, Ann. Appl. Probab., 2008]. This talk is based on a joint work in progress with Yu Liu (PKU) and Yuanzheng Wang (PKU).

Sharp bounds on the least positive harmonic measure in lattices

Eviatar Procaccia (Technion)

We discuss the least positive harmonic measure for sets in $\mathbb{Z}^d$ and planar lattices. For planar lattices we present the optimal asymptotic rate and we conjecture the sets achieving it are $*$-connected spirals. For $\mathbb{Z}^d$, $d > 2$, we do not know whether the optimal sets are $*$-connected, thus we consider scattered sets with some minimal distance between points of the set. We prove sharp bounds that present a phase transition in the minimal distance parameter. Various open problems will be presented. Based on joint work with Zhenhao Cai, Yam Berent, Gady Kozma and Yuan Zhang.

TUESDAY

Loop-erased random walk in three dimensions

Daisuke Shiraishi (Kyoto U.)

Loop-erased random walk (LERW) is a random simple path obtained through erasing all loops chronologically from a simple random walk path. In this talk, I will discuss recent progress on the convergence of 3D LERW in the natural parametrization. This is joint work with Xinyi Li and Sarai Hernandez-Torres.

Minkowski content of the scaling limit of 3D loop-erased random walk

Saraí Hernández-Torres (UNAM, Mexico)

This talk will present a closer look at the proof of the existence of the Minkowski content of the scaling limit of the 3D loop-erased random walk (LERW), following Daisuke Shiraishis talk. Due to the absence of essential tools in the continuum, crucial aspects of the analysis take place in discrete space. Specifically, we obtain sharp estimates for the one-point function and ball-hitting probabilities of the LERW on $\mathbb{Z}^3$. This work is a collaboration with Xinyi Li and Daisuke Shiraishi.

Cluster sizes for the vacant sets of random walk on a torus

Subhajit Goswami (Tata Institute, Mumbai)

Consider a simple random walk on a discrete torus of large side length N in dimensions ≥ 3 starting from a uniform point. When the walk runs for a time uN^d for some $u > 0$, the corresponding vacant set, i.e. the set of points not visited by the walk, undergoes a percolation phase transition across a value $u_* \in (0, \infty)$. In this talk we will discuss some recent results on the largest and second largest diameter of a vacant cluster in the subcritical ($u > u_*$) and supercritical ($u < u_*$) regime respectively. Interestingly, both these diameters grow at rate $\log N \log\log N$ in dimension 3 whereas in all higher dimensions they grow at a slightly slower, albeit "more familiar", rate $\log N$. Furthermore in dimension 3, we can compute the precise prefactor in front of $\log N \log\log N$ which turns out to depend symmetrically on u on either side of u_* and diverges polynomially as $u \to u_*$. Based on upcoming works with Pierre-Francois Rodriguez and Yuriy Shulzhenko.

On the cover time of Brownian motion on the Brownian continuum random tree

David Croydon (RIMS, Kyoto U.)

Upon almost-every realisation of the Brownian continuum random tree (CRT), it is possible to define a canonical diffusion process or 'Brownian motion'. I will discuss a recent result that establishes the cover time of the Brownian motion on the Brownian CRT (i.e. the time taken by the process in question to visit the entire state space) is equal to the infimum over the times at which the associated local times are strictly positive everywhere. The proof of this result depends on the recursive self-similarity of the Brownian CRT and a novel version of the first Ray-Knight theorem for trees, which is of independent interest. As a consequence, it can be deduced that the suitably-rescaled cover times of simple random walks on critical, finite variance Galton-Watson trees converge in distribution with respect to their annealed laws to the cover time of Brownian motion on the Brownian CRT. Other families of graphs that have the Brownian CRT as a scaling limit are also covered. Additionally, the result partially confirms a 1991 conjecture of David Aldous regarding related cover-and-return times. This project is joint with George Andriopoulos (NYU Abu Dhabi), Vlad Margarint (Charlotte) and L. Menard (Paris Nanterre).

Scaling limit of high-dimensional random spanning trees

Eleanor Archer (U. Paris Nanterre)

A spanning tree of a finite connected graph G is a connected subgraph of G that touches every vertex and contains no cycles. In this talk we will consider uniformly drawn spanning trees of high-dimensional graphs, and explain that, under appropriate rescaling, they converge in distribution as metric-measure spaces to Aldous Brownian CRT. This extends an earlier result of Peres and Revelle (2004) who previously showed a form of finite-dimensional convergence. If time permits, we may also discuss scaling limits of random spanning trees with non-uniform laws. Based on joint works with Asaf Nachmias and Matan Shalev.

A (dis)continuous percolation phase transition on the hierarchical lattice

Johannes Bäumler (UCLA)

For long-range percolation on $\mathbb{Z}$ with edge kernel J, it is a classical theorem of Aizenman and Newman (1986) that the phase transition is discontinuous when $J(x-y)$ is of order $|x-y|^{-2}$ and that there is no phase transition at all when $J(x-y) = o(|x-y|^{-2})$. We discuss analogous theorems for the hierarchical lattice, where the relevant threshold is at $|x-y|^{-2d} \log \log |x-y|$ rather than $|x-y|^{-2}$: There is a continuous phase transition for kernels of larger order, a discontinuous phase transition for kernels of exactly this order, and no phase transition at all for kernels of smaller order. Joint work with Tom Hutchcroft.

An invariance principle for de-randomized conductance models

Marek Biskup (UCLA)

The theory of random walks in disordered reversible environments, a.k.a. conductance models, has advanced remarkably under the assumption that the environment is drawn at random from a law that is stationary and ergodic under translates. Indeed, the concept of the "point of view of the particle" enables ergodic theorems that conveniently produce limits that would be difficult to establish otherwise; the corrector method from stochastic homogenization then yields a proof of diffusive scaling to Brownian motion. Unfortunately, the fact that both the input and the statement are stochastic make it all but impossible to decide whether the resulting invariance principle holds for any given (non-periodic) conductance configuration. I will show how to overcome this by de-randomizing both the mixing theory for the "point of view of the particle" and the corrector method. An invariance principle will then hold for *every* conductance configuration (modulo certain growth/decay restrictions) whose block averages converge and define an ergodic law on the space of environments.

Activation times for the long range frog model

Omer Angel (UBC)

We consider the frog model on either $\mathbb{Z}^d$ or the torus. At each site there are initially a Poisson number of sleeping frogs. When an active frog visits a site, all frogs there wake up, perform a random walk for some time, and die. In this work, our random walks are heavy tailed. We connect this model with long range percolation, and derive estimates for the hitting time of a vertex and the cover time of the torus. Joint with Jonathan Hermon and Yuliang Shi.

The random Lorentz gas – invariance principle beyond the kinetic time scales

Bálint Tóth (U. Bristol & Rényi Inst., Budapest)

Since the pioneering work of Paul and Tatiana Ehrenfest (1912) the deterministic (Hamiltonian) motion of a point-like particle exposed to the action of a collection of fixed, randomly located short range scatterers has been a much studied model of physical diffusion under fully deterministic (Hamiltonian) dynamics, with random initial conditions. This model of physical diffusion is known under the name of "random Lorentz gas" or "random wind-tree model. Celebrated milestones on the route to better mathematical understanding of this model of true physical diffusion are the Kinetic Limits for the tagged particle trajectory under the so-called Boltzmann-Grad (a.k.a. low density), or weak coupling approximations [Gallavotti (1970), Spohn (1978), Boldrighini-Bunimovich-Sinai (1982), respectively, Kesten-Papanicolaou (1980)]. Once the kinetic limits are established, under a second diffusive space-time scaling limit the central limit theorem (CLT) and invariance principle (IP) for the tagged particle motion follow. However, the CLT/IP under bare diffusive space-time scaling (without first applying the kinetic approximations) remains a Holy Grail.

In recent work we have obtained some intermediate results, partially interpolating between the two-steps-limit (first kinetic, then diffusive - as described above) and the bare-diffusive-limit (Holy Grail). We establish the Invariance Principle for the tagged particle trajectories under a joint kinetic+diffusive limiting procedure, performed simultaneously rather than successively, reaching significantly longer time scales than any earlier result (like, e.g., Komorowski-Ryzhik (2006) in classical, or Erds-Salmhofer-Yau (2008) in quantum weak coupling setting). The main ingredient is a coupling of the Hamiltonian trajectory (with random initial conditions) and an approximate Markovized version of the motion, and probabilistic and geometric controls on the efficiency of this coupling.

The Holy Grail remains, however, beyond reach.

THURSDAY

AC conduction in disordered media

Alessandra Faggionato (La Sapienza)

Transport in disordered media under the effect of a time-oscillating external field is relevant in many applications. We focus here on the oscillatory steady state of the system for weak fields. The linear response of the mean Alternating Current can be described by the complex mobility matrix. We derive its form for RWs and diffusions and show, via 2-scale stochastic homogenization, its a.s. limit for random conductance models in a periodized environment as the spatial period size diverges. We then discuss the abstract structure underlying linear response of the oscillatory steady state for a large class of observables.

Directed polymers on supercritical percolation clusters

Maximilian Nitzschner (HKUST)

In this talk, we introduce a model of directed polymers on infinite clusters of supercritical Bernoulli percolation containing the origin in dimensions $d \geq 3$. We prove that for almost every realization of the cluster and every strictly positive value of the inverse temperature, the polymer is in a strong disorder phase, answering a question from Cosco, Seroussi, and Zeitouni. The proof is based on a utilization of a fractional moment calculation, as well as a large deviation-type estimate on the number of effectively one-dimensional tubes present in the intersection of a large box and the supercritical cluster.

Equivalence of strong disorder and very strong disorder for directed polymers

Stefan Junk (Gakushuin U.)

We consider the directed polymer model, which describes paths affected by a random space-time environment. In spatial dimension $d > 2$, the model undergoes a phase transition between a high-temperature, weak disorder phase and a low-temperature, strong disorder phase, which is characterized by whether the partition function converges to zero. In strong disorder, certain localization properties of the model are only known if we require that the partition function converges to zero even exponentially fast, which is known as the very-strong-disorder regime. It has been a long-standing conjecture that these two notions are equivalent and we now give a proof of this. Moreover, our proof reveals that the critical value itself belongs to the weak disorder phase. Joint work with Hubert Lacoin.

Arm exponents for metric graph Gaussian free fields

Zhenhao Cai (Peking U.)

The metric graph GFF is a natural extension of the discrete GFF. This talk will introduce our recent progress on the estimates for the critical one-arm probability (namely, the probability that the origin is connected to the boundary of a large box by the critical level-set). This is a joint work with Jian Ding.

Percolation of hierarchical interlacement on $\mathbb{Z}^d$

Balázs Ráth (BME & Rényi Inst., Budapest)

Hierarchical random walk is a Markov chain with state space $\mathbb{Z}^d$ which shares some properties with long-range random walks, e.g. polynomial tail decay of the Green function in the transient case. We consider random interlacements on $\mathbb{Z}^d$ based on hierarchical random walk. We show that the interlacement set (a) undergoes non-trivial percolation phase transition as the intensity varies for some choices of the model parameters, while (b) it percolates at all positive intensities for other choices of the model parameters (e.g., for the latter result we have to assume that d is at least four). Joint work in progress with Sándor Rokob.

FRIDAY

Poisson-Dirichlet distribution for the interchange process in five dimensions

Dor Elboim (IAS)

In the interchange process on a graph $G = (V, E)$, distinguished particles are placed on the vertices of G with independent Poisson clocks on the edges. When the clock of an edge rings, the two particles on the two sides of the edge interchange. In this way, a random permutation $\pi_\beta : V \to V$ is formed for any time $\beta > 0$. One of the main objects of study is the cycle structure of the random permutation and the emergence of long cycles. We consider the process on the torus of side length L in dimension $d \geq 5$ and prove that macroscopic cycles emerge after a long time β. These are cycles whose length is proportional to the volume of the torus L^d. Moreover, we show that the cycle lengths converge to the Poisson-Dirichlet distribution. This is a joint work with Allan Sly.

Random walk loops, dimers, spins and the Bose gas

Lorenzo Taggi (U. La Sapienza)

We introduce a random walk loop soup which reduces to or is related to several models of interest in statistical mechanics, including the Spin $O(N)$ model, the Bose gas, the dimer model, the double dimer model, random lattice permutations. The main questions involve the characterisation of the size and the geometry of the random walk loops, which interact by mutual repulsion. We present some results about the existence of macroscopic loops in three and higher dimensions and some more recent results on the absence of macroscopic loops in two dimensional (not necessarily planar) graphs.

A conjectured percolation inequality

Gady Kozma (Weizmann I.)

We will discuss why the titular inequality is interesting, and why we believe it to be correct. Joint work with Shahaf Nitzan.