DFG SPP2265 Summer School on

# Particle systems in random environments

Organisers: Alice Callegaro (TU Munich),
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Frankfurt, August 26-30, 2024

Funded by the DFG through the Schwerpunktprogramm 2265 *Random Geometric Systems.*

# Contents

# 1 Programme

| | Monday | Tuesday | Wednesday | Thursday | Friday |
|---|---|---|---|---|---|
| 8:30-9:00 | Registration | | | | |
| 9:00-10:00 | Valesin | Gantert | Avena | Valesin | Gantert |
| 10:15-11:15 | Valesin | Gantert | Avena | Valesin | Gantert |
| 11:15-11:45 | Coffee break | | | | |
| 11:45-12:30 | Terveer | Boenkost | Quattropani | van de l'Isle | Archer |
| 12:30-14:00 | Lunch break | | | | |
| 14:00-15:00 | Avena | Valesin | Gantert | Avena | |
| 15:15-16:15 | Avena | Valesin | Gantert | Avena | |
| 16:15-16:45 | Coffee break | | | | |
| 16:45-17:30 | Poster session | Fernley | Schweiger | van Belle | |
| from 19:00 | | | Conference Dinner | | |

# 2 Locations and campus map

The **lectures** will take place in the lecture hall IV (HIV), which is situated in the Hörsaalgebäude (15) and the **Poster session** will take place in the foyer of the Neue Mensa (5).

The **conference dinner** will take place at *Restaurant Feldbergblick*, Ginnheimer Stadtweg 57A, 60431 Frankfurt am Main. For directions, see `https://maps.app.goo.gl/maTYnggqeGWTvA9v9`


Map of Campus Bockenheim

# 3 Lunch recommendations

1. **Neue Mensa**, no. 5 on campus map (cheap, fast)
2. **Heppy Green**, Jordanstraße 1 (healthy food)
3. **Isoletta**, Schloßstraße 126 (Italian)
4. **Lillium**, Leipziger Straße 4 (divers)
5. **Mangetsu**, Varrentrappstraße 57 (Japanese)
6. **Namaste India**, Jordanstraße 6 (Indian)
7. **Onkel Lam**, Schloßstraße 123 (Vietnamese)

There are lots of further options closeby, e.g. further down the Leipziger Straße.

# 4 Minicourses

## Luca Avena (University of Florence)

### Random Walks in Dynamic Random Environment on $\mathbb{Z}^d$

The theory of homogeneous random walks on the $d$-dimensional integer lattice represents one of the cornerstones of probability theory. Over the last 50 years major efforts have been devoted to extend this theory to models of random walks for which the underlying Markov transition kernel is non-homogeneous in space and time. Such models have in fact natural applications in various fields ranging e.g. from particle physics, to biology and finance. One fruitful way to analyse such a discrete random motion in heterogeneous media consists of considering a space-time random field on the integer lattice, the so-called random environment, and letting the transition kernel of the random walk be a function of this random environment. This neat mathematical modelling framework has led to many new questions and, depending on the law of the underlying random environment, to new results and behaviour. In fact the presence of a medium with random impurities brings non-trivial correlated structures which represent a serious mathematical challenge, and may give rise to new localisation effects due to geometrical trapping phenomena for which the long run statistics of the walk may differ or not with respect to the classical homogeneous setup.

The goal of the lectures is to offer an overview on limiting results (e.g. recurrence, asymptotic speed, invariance principle, large deviations), techniques and current research directions in this area at least for certain classes of models for which we have been gradually witnessing the emergence of several new frameworks and toolboxes. The rough plan of the 6 lectures is as follows:

Lecture 1- Historical account on various classes of static and dynamic models and some showcase examples.

Lecture 2- Homogenisation theory & the environment as seen from the particle: martingale and Kipnis-Varadhan approaches for reversible and nearly reversible models.

Lecture 3- Homogenisation theory & the analysis of the walk trajectories: cut-points and regeneration times, space-time “decorrelation” techniques in static and dynamic environments.

Lecture 4- Localisation theory in $1d$ for static environments: classical potential theory.

Lecture 5- Localisation theory in $1d$ for dynamic environments: cooling random environments.

Lecture 6- Localisation theory for dynamic environments: cooling and other problems.

## Nina Gantert (Technical University of Munich)

### Branching random walks

Lectures 1-2: Definition of the model, and definition of the more general branching Markov chain model. Recurrence and transience of Branching Markov Chains. Some remarks on the frog model.

Lectures 3-4: Extremal values of branching random walks, linear growth of the position of the rightmost particle.

Lectures 5-6: More on extremal values, the second term in the growth of the position of the rightmost particle. The Dekking-Host argument for tightness.

References:

Zhan Shi: Branching Random Walks.
Yuval Peres: Probability on Trees: an Introductory Climb.
Russell Lyons and Yuval Peres: Probability on Trees and Networks.
Ofer Zeitouni: Branching Random Walks and Gaussian Fields.

## Daniel Valesin (University of Warwick)

### The contact process on dynamic graphs

In this lecture series, we study recent developments on the contact process on evolving graphs. The contact process is a simple model for the spread of an infection in a population. In the last two decades, there has been growing interest on the behavior of this process on random graph models which reflect real-world populations. An extra layer of realism and mathematical complexity comes from allowing the graph to evolve simultaneously with the infection, and a rapidly growing body of work has been dedicated to this sort of model. After giving a brief overview of the theory of the classical contact process, we focus on two recent developments on dynamical graphs. The first of these is the study of the contact process on dynamical percolation on the Euclidean lattice. This is an appealing extension of the classical contact process, with many interesting phenomena arise, for instance the occurrence of parameter regimes where the graph is immune to the infection, no matter how high the infection rate. The second is the contact process on dynamic d-regular graphs with an edge-switching dynamics. The highlight of this setting is a strict monotonicity result, that shows that the graph dynamics strictly helps the infection to spread.

# 5 Invited talks

## Eleanor Archer (Université Paris Nanterre)

### Quenched critical percolation on Galton-Watson trees

We consider critical percolation on a supercritical Galton-Watson tree with mean offspring $m > 1$. It is well known that the critical percolation probability for this model is $1/m$ and that the root cluster has the distribution of a critical Galton-Watson tree. For this reason, many properties of the cluster are well understood, for example the probability of surviving for at least $n$ generations, the limiting law of the size of the $n$-th generation conditioned on survival (the "Yaglom limit"), and convergence of the entire cluster to a branching process/stable tree. All these results are annealed, that is, we take the expectation with respect to the distribution of the tree and the percolation configuration simultaneously. The goal of this talk is to consider the quenched regime: are the same properties true for almost any realization of the tree? We will see that this is indeed the case, although some scaling constants will depend on the tree.
Based joint work with Quirin Vogel and an ongoing project with Tanguy Lions.

## Florin Boenkost (Universität Wien)

### The genealogy of nearly critical branching processes in a varying environment

Branching processes in varying environment (BPVE) are a natural generalisation of classical Galton-Watson processes allowing for a time inhomogeneous offspring distribution. In this talk we introduce a notion of *near-criticality* for BPVEs and discuss Kolmogorov's asymptotics, Yaglom's law and the genealogical structure of nearly-critical BPVEs. Building on the spinal decomposition in [Foutel-Rodier, Schertzer, 22] we prove that the genealogy seen as a random metric space converges in the Gromov-Hausdorff-Prohorov topology to a time-changed Brownian coalescent point process. Through this convergence result we are able to deduce Yaglom's law, the time to the most recent common ancestor and convergence of the reduced process. This talk is based on joint work with Félix Foutel-Rodier and Emmanuel Schertzer.

## John Fernley (Alfréd Rényi Institute of Mathematics)

### The phase transition of the voter model on evolving scale-free networks

The voter model is a classical interacting particle system explaining consensus formation on a social network. Real social networks feature not only a heterogeneous degree distribution but also connections changing over time. We study the voter model on a rank one scale-free network evolving in time by each vertex *updating* (refreshing its edge neighbourhood) at any rate $\kappa = \kappa(N)$.

We find the dynamic giant component phase transition in the consensus time of the voter model: when $\kappa \ll 1/N$, the subcritical graph parameters are slower by a factor of $N/\log N$. Conversely, when $\kappa \gg 1$ the effect of the giant is removed completely and so for either graph parameter case we see consensus time on the same order as in the static supercritical case (up to polylogarithmic corrections). The intermediate dynamic speeds produce consensus time for subcritical network parameters longer not by the previous factor $N/\log N$, but by the factor $1/\kappa$.

## Matteo Quattropani (La Sapienza, University of Rome)

### Meeting and consensus time on random digraphs

We consider the two-opinion voter model on a random directed graph with prescribed in- and out-degree distributions. The configuration is initialized with a Bernoulli product measure of parameter p on the vertex set, and each vertex updates its opinion at rate 1 by copying the opinion of one of its out-neighbors uniformly at random. It is known that, under mild conditions the density of vertices with a given opinion converges to a Wright-Fisher diffusion after a proper time rescaling, corresponding to a time acceleration by the expected meeting time of two independent random walks started at equilibrium. Moreover, a first order approximation of the expected meeting time can be translated into a first order approximation of the consensus time of the voter model. For this reason, we analyze the expected meeting time of two independent walks on a typical realization on the digraph, and show that it concentrates around a deterministic quantity depending only on the degree sequences, for which we provide an explicit formula. The talk is based on a joint work with Luca Avena (Firenze), Federico Capannoli and Rajat Hazra (Leiden).

## Florian Schweiger (University of Geneva)

### Extrema of log-correlated Gaussian fields in random environment

Gaussian fields can be thought of as stochastic models for interfaces. They share many features with branching random walks. I will explain this connection, and then describe some results about the extrema of these models in deterministic and random environment.

## Sara Terveer (LMU Munich)

### Spectral properties of the stochastic block model and their application to hitting times of random walks

Random walks are a powerful tool to analyze properties of random graphs. Hitting times are one of many characterizing quantities in this context. Recently, many results for hitting times in the Erdős–Rényi model have been established. In this talk, we are going to discuss a closely related model, the stochastic block model, in terms of spectral properties of its (symmetrically normalized) adjacency matrix. The results are then applied to the averaged hitting times to obtain asymptotics. Joint work with Matthias Löwe.

## Thomas van Belle (Universität Duisburg Essen)

### Meeting times of random walks via the singular value decomposition

We suggest a non-asymptotic matrix perturbation approach to get sharp bounds on the expected meeting time of random walks on large (possibly random) graphs. We provide a formula for the expected meeting time in terms of the singular value decomposition of the diagonally killed generator of a pair of independent random walks, which we view as a perturbation of the generator. As the proof of concept, we work out sharp bounds on the expected meeting time of simple random walks on sufficiently dense Erdős–Rényi random graphs.

## Carmen van de l'Isle (University of Bath)

### The symbiotic contact process on Galton-Watson trees

The symbiotic contact process can be thought of as a two type generalisation of the contact process which can be used to model the spread of two symbiotic diseases. Each site can either be infected with type A, type B, both, or neither. Infections of either type at a given site occur at a rate of lambda multiplied by the number of neighbours infected by that type. Recoveries of either type at a given site occur at rate 1 if only one type is present, or at a lower rate mu if both types are present, hence the symbiotic name. Both the contact process and the symbiotic contact process have two critical infection rates on a Galton-Watson tree, one determining weak survival, and the other strong survival. Here, weak survival refers to the event where at least one A infection and at least one B infection is present at all times. Strong survival is the event that the root of the tree is infected with both A and B infections at the same time infinitely often. In this talk, I will prove that for small values of mu the weak critical infection rate for the symbiotic model is strictly smaller than the critical rate for the contact process. I will also discuss the more complicated case of strong survival for both processes.