Technische Universität München
Zentrum Mathematik

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Winter term 2020/21
Exercise sheet 8

Abelian varieties (MA 5115)

As usual, we work over an algebraically closed field k.
This week are considering quotients of schemes by finite groups. As we will see next week, if we form the quotient of an Abelian variety by a finite group, the quotient is an Abelian variety and the quotient map is an isogeny. In other words, the quotient construction allows us to construct isogenies (if $\operatorname{char} k = 0$ even "all of them").

Exercise 1. Let X be a scheme with a finite group G acting on it such that the quotient $X /\!\!/ G$ exists. We want to show the following universal property. Let $\pi\colon X \to X'$ be a G-invariant morphism of schemes (i.e.$\pi(gx) = x$ for all g, x). Then there exists a unique morphism $X /\!\!/ G \to X'$ such that the diagram

$$\begin{array}{ccc} X & \xrightarrow{\pi'} & X' \\ \downarrow & \nearrow & \\ X /\!\!/ G & & \end{array}$$

commutes.

(a) First show that there exists a unique continuous map of topological spaces such that the diagram is commutative.

(b) now show that there exists a unique morphism of schemes

Exercise 2. In order to define quotients of varieties, we are going to need the following property. If X is a quasi-projective variety $S \subset X$ a finite set, then there exists $U \subset X$ affine open such that $S \subset U$.

(a) Prove the statement for projective varieties (as we will apply it to Abelian varieties)

(b) Prove the statement in general.

Deadline: Monday, 18th January, 2011